The “Modeling & Analysis Competency Demonstration Project” is designed to assess your ability to use the skills in linear modeling, linear systems & programming, and counting techniques that you’ve learned in a practical real-world application. The project scenario is a small-business startup and does not focus on EVERY aspect of a small business startup, but does focus on cost optimization and profit/loss calculation and prediction through the use of linear modeling and linear programming.

**Instructions**:

1. Access (and download if you like) the scenario (in either MS Word of PDF format) from **Content > Competency Demonstration Projects > Modeling & Analysis Competency Demonstration Project** in our LEO classroom.
2. Perform each of the 10 required tasks to the best of your ability. Note that some of the tasks require calculations, others require linear equation construction, still others require graphing. PLEASE show all your work when doing calculations, equation construction and graphing if you want me to consider awarding partial credit! I cannot give partial credit if you don’t show your work!
3. Compile your submission. Be sure your submission includes:
   1. Answers to the questions posed in each of the 10 tasks;
   2. Required graphs (2) and whatever supporting graphs you wish to include
   3. Whatever work supporting your answers that you wish me to consider for partial credit (of course, if you don’t submit any work and your answer is incorrect, then I’m obliged to award no credit for your answer…)
4. Upload your submission into the “Modeling & Analysis Competency Demonstration Project” folder in the *Assignments* link under the NavBar in our LEO classroom. **Be sure to forward your initial submission no later than 11:59PM Eastern time on the Sunday ending the 5th week of class** if you want to be eligible for full credit for this project! Any initial submissions received after this deadline are only eligible for a maximum of 75 out of 100 points (see “Grading Criteria” section of **Content > Course Syllabus** for more details).
5. You’ll receive an initial grade and feedback (grade will be visible in your Gradebook; grade and feedback will be visible in your Assignment Folder). You are free (and encouraged!) to make changes to your submission based on the feedback you receive and then resubmit your project for a better score. You may do this as many times as you wish prior to 11:59PM Eastern time on the Thursday of Week 8. I will grade resubmissions and post your updated project grade as well as updated feedback (if necessary).

If you have any questions about this project, please post them in the “Modeling & Analysis Competency Demonstration Project Discussion” topic found at **Discussions > Modeling & Analysis Competency Demonstration Project** forum in our LEO classroom so everyone can see what we’re talking about. Good luck on the project!

**Business Case Study – Homer’s Donuts**

Homer’s Donuts is a small-business startup looking to open its first franchise location in Smallville. Your ownership team completed its business and marketing SWOT (“strengths – weaknesses – opportunities – threats”) analyses and believes there’s a real chance for Homer’s Donuts to be a successful business!

The SWOT marketing analysis included a survey of potential customers in and around the Smallville metropolitan area. The survey asked people what pastries they would most likely buy when they stop for coffee on the way to wherever they were going. 2000 people responded to the survey, and the results are as follows:

* 1415 respondents said they would buy glazed donuts
* 1605 respondents said they would buy creme-filled frosted donuts
* 180 respondents said they would buy neither glazed nor creme-filled frosted donuts

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**Task 1: Interpreting the Marketing Survey: Counting**

For the survey results above:

1. Create either a Venn diagram or a contingency table depicting the results of the survey.
2. Determine how many of the survey respondents would buy either a glazed donut or a creme-filled frosted donut?
3. Determine how many of the survey respondents would buy a glazed donut and a creme-filled frosted donut?

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From your SWOT analyses, your team decides the best way to go is to offer assorted donuts, sold individually or by the dozen. Now your team has to come up with a daily operation cost/revenue analysis to see if it’s actually worth going into business. Here’s what the team knows so far:

1. Before making any donuts, there is a $770 fixed cost each day (which covers the building and its equipment, as well as labor costs for 2 shifts of 4 employees each). That means the cost to produce 0 dozen assorted donuts per day is $770. The cost to produce 30 dozen assorted donuts per day is $824; the cost to produce 45 dozen assorted donuts per day is $851; and the cost to produce 60 dozen assorted donuts per day is $878. The relationship between dozens of donuts produced per day and cost per day is linear.
2. To undercut the competition, your team recommends setting the price per dozen assorted donuts so that Homer’s gets $440 total (gross) income for 50 dozen assorted donuts sold per day, and $660 total (gross) income for 75 dozen assorted donuts sold per day. The relationship between dozen donuts sold per day and total (gross) income per day is linear.

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**Task 2**: **Determine Cost Equation**

Use the information in paragraph A above to:

* Create at least two ordered pairs. Let the *x* value be the number of dozen donuts produced per day and the *y* value be the cost to produce them.
* Use two of the ordered pairs to determine the slope *m* of the linear equation that describes daily operating cost *C(x)* where *x* represents number of dozens of donuts produced per day. Interpret the meaning of the slope.
* Determine the *y* – intercept value (also known as *b*)
* Use what you’ve found to create the linear equation in *y = mx + b* (slope-intercept form) that describes daily production costs for Homer’s Donuts
* Take the equation you created and replace “*y*” with “*C(x)*” to create the **cost equation**.

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**Task 3**: **Determine Revenue Equation**

Use the information in paragraph B above to:

* Create at least two ordered pairs. Let the *x* value be the number of dozen donuts produced per day and the *y* value be the total income (revenue) from selling them.
* Use two of the ordered pairs to determine the slope *m* of the linear equation that describes daily revenue *R(x)* where *x* represents number of dozens of donuts produced per day. Interpret the meaning of the slope.
* Determine the *y* – intercept value (also known as *b*)
* Use what you’ve found to create the linear equation in *y = mx + b* (slope-intercept form) that describes daily revenue for Homer’s Donuts
* Take the equation you created and replace “*y*” with “*R(x)*” to create the **revenue equation**.

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**Task 4: Graph Cost and Revenue Equations on Same Chart**

Using the information you created in Tasks 2 and 3, create a single chart that shows:

1. Ordered pair points for daily production cost you created in Task 2
2. Ordered pair points for daily revenue you created in Task 3
3. Graph of the cost equation you created in Task 2
4. Graph of the revenue equation you created in Task 3

Use an appropriate scale on the horizontal and vertical axes of your chart, make sure the correct values go with the correct axis, and be sure to label carefully (chart title, axes titles, etc). You may create your chart using software (MS Excel, DESMOS, etc) or you may draw it by hand. This chart will help you answer task 5 requirements, so do your best to make it accurate!

* If you use Microsoft Excel, be sure to check out **Content > Course Resources > Webliography** in our LEO classroom for two topics on scatterplot construction and linear regression which are very helpful when creating a chart.
* If you want to use the free DESMOS graphing software to create your chart, be sure to check out **Content > Course Resources > Webliography > “Desmos” Free Online Graphing Calculator** in our LEO classroom to access DESMOS, make your chart on that site, and save the image to upload into your project submission. Be sure it’s labeled correctly!
* If you draw by hand, the chart MUST be proportionally accurate (that means use graph paper as a minimum), all points and graphed lines must be identified, and axes and scales must be correctly labeled.

When completed, your chart depicts the “cost/revenue model” for Homer’s Donuts!

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Your team knows that Homer’s Donuts has to sell a minimum number of dozens of assorted donuts every day before it starts to make a profit. That minimum number of dozens of assorted donuts needing to be sold every day is defined as the *break-even point.*

**Task 5: Break-Even Point Determination**

Identify the number of dozens of assorted donuts *x* that need to be sold per day in order for daily revenue to equal daily production costs [which means that value of *x* which makes *R(x) = C(x)* ]. Also identify the total daily cost and total daily revenue at the break-even point. You may determine the break-even point graphically (using chart you created in Task 4) or algebraically by making a system of the two linear equations you created in Tasks 2 and 3 and solving that system. *Answers must be exact* !

Break-even occurs when *x* = \_\_\_\_\_\_\_\_\_\_\_\_\_ dozens of assorted donuts are sold daily.

At break-even point, *C(x) =*  $ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ per day

At break-even point, *R(x) =*  $ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ per day

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**Task 6: Using Model to Determine Performance and Net Operating Revenue (Profit or Loss)**

Using the “cost/revenue model” graph created in Task 4 and/or the equations created in Tasks 2 and 3, determine how Homer’s Donuts would perform by completing the following table (*fill in all the blanks – give answers rounded to the nearest dollar*):

|  |  |  |  |
| --- | --- | --- | --- |
| If # dozen assorted donuts sold daily *x* = | Then total daily production cost *C(x)* (in $) = | And daily total (gross) revenue *R(x)* (in $) *=* | And net daily operating revenue *R(x) – C(x)* (in $) = |
| 80 |  |  |  |
| 110 |  |  |  |
| 200 |  |  |  |
| 500 |  |  |  |

For net daily operating revenue: Positive values represent PROFIT; negative values represent LOSS

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Now it’s time to look at the advertising budget for Homer’s Donuts!

The advertising agency you hired presented the following costs for Internet and television ads for Homer’s Donuts. Producing an Internet ad costs $400 and producing a television ad costs $600. The cost to run an Internet ad is $1000, and it will cost $6000 to run a TV ad. Your team decides that the total amount to be spent on ads is not to exceed $4,800 for production and not to exceed $30,000 for running the ads. The agency estimates that each viewing of an Internet ad will reach 1000 people and each airing of a television ad will reach 1600 people. How many of each type of ad should be bought so that the total number of people reached is a maximum?

Linear programming is the mathematical method best used to determine the values of input variables needed to optimize the “objective function”, whether that function is maximizing the number of people an ad campaign reaches or minimizing the costs associated with the operation of a manufacturing process.

**Task 7: Identify Constraints on the Process**

* From the scenario, identify what variables *x* and *y* will represent.
* Using the information above fill in the following table, and then use the table to create the two algebraic equations or inequalities that identify the constraints on the process of advertising for Homer’s Donuts:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Ad Process Task | Cost per Internet ad ($) | # of Internet ads | Cost per TV ad ($) | # of TV ads | Enter correct sign ( | Total Cost ($) |
| Production |  |  |  |  |  |  |
| Running |  |  |  |  |  |  |

Constraint on producing ads: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Constraint on running of ads: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

If the variables *x* and *y* cannot be negative, be sure to include and as two additional constraints!

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**Task 8: Identify Objective Function**

Algebraically define the objective function, which is the “reach” *R* that this ad campaign will have. Fill in the following table, and then use the table to create the algebraic “objective function” that defines the reach *R* (in terms of *x* and *y*) for Homer’s Donuts advertising campaign:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| # people reached by Internet ad | # of Internet ads | Enter correct sign (+, – ) | # people reached by TV ad | # of TV ads | Enter correct sign ( | Reach (*R*) |
|  |  |  |  |  |  | *R* |

Algebraic form of objective function: *R* = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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**Task 9: Graph Constraints, Identify Feasible Region and Corner Points**

Using the constraints you defined in Task 7, construct a graph with the following features identified:

* All constraint linear equations/inequalities graphed
* “Feasible region” with its boundaries
* Coordinates of each “corner point” of the feasible region

Use an appropriate scale on the horizontal and vertical axes of your chart, make sure the correct values go with the correct axis, and be sure to label carefully (chart title, axes titles, etc). You may create your chart using graphing calculators (DESMOS, TI-80 series graphing calculators, etc) or you may draw it by hand. Do your best to make it accurate!

* If you want to use the free DESMOS graphing software to create your chart, be sure to check out **Content > Course Resources > Webliography > “Desmos” Free Online Graphing Calculator** in our LEO classroom to access DESMOS, make your chart on that site, and save the image to upload into your project submission. Be sure it’s labeled correctly!
* If you draw by hand, the chart MUST be proportionally accurate (that means use graph paper as a minimum), all points and graphed lines must be identified, and axes and scales must be correctly labeled.

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**Task 10: Optimizing the Objective Function**

Using the objective function determined in Task 8 and the graph created in Task 9:

* Create a corner point table by filling in the blanks of the following table (enter as many rows as you need). For each corner point entered in the left column, evaluate the objective function by entering the corresponding corner point values for *x* and *y* and solve for *R*. Enter value of *R* in right column

|  |  |
| --- | --- |
| Corner Point Coordinates (*x , y*) | Enter objective function:  *R* = |
|  |  |
|  |  |
|  |  |
|  |  |

* Determine which corner point coordinate pair optimizes the objective function:
  + If you want to maximize *R,* select the corner point pair thatproduces the greatest value of *R*
  + If you want to minimize *R*, select the corner point pair that produces the smallest value of *R*.

The corner point pair you select represents the number of Internet ads to be created and the number of TV adds to be created to result in maximum “reach” *R*. You cannot “mix” corner point values; you must use the *x* and *y* values of the corner point you select.

Number of Internet ads that should be bought to maximize “reach” *R* : \_\_\_\_\_\_\_\_\_\_\_\_

Number of times TV ads that should be bought to maximize “reach” *R* : \_\_\_\_\_\_\_\_\_\_\_

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